Statistical Mechanics of Lattice Systems

by Paolo Rinaldi

Course Diary

- 10/10/23: Introduction to the course; brief survey of equilibrium thermodynamics; introduction to statistical mechanics, microcanonical ensamble, Shannon entropy and the maximum principle.
- 11/10/23: Canonical ensamble; canonical Gibbs measure; magnetic systems and their phenomenology; paramagnet and ferromagnet; introduction to the Ising model; Hamiltonian of the Ising model and its symmetries; introduction to the thermodynamic behaviour.
- 17/10/23: Thermodynamics behaviour; analysis of the limits for $\beta \to 0$ and for $\beta \to +\infty$; analysis of the scenario with large but finite β in the limit of $n \to +\infty$; Finite volume Gibbs distribution; Boundary condition for the Ising model: free boundary, periodic boundary conditions and specific boundary conditions; definition of convergence to $\mathbb{Z}^d$ of family of subsets $\{\Lambda_n\}_{n\in\mathbb{N}} \subset \mathbb{Z}^d$ and convergence in the sense of Van Hove.
- 18/10/23: Definition of pressure; convexity of the pressure as a function of β and h; existence of the thermodynamic limit of the pressure and its independence from the chosen sequence $\{\Lambda_n\}_{n\in\mathbb{N}} \subset \mathbb{Z}^d$ and from the chosen boundary conditions (proof not completed).
- 24/10/23: Conclusion of the proof started in the previous lecture; definition of total magnetization and magnetization density; relation between average magnetization density and pressure for finite region; thermodynamic limit of the average magnetization and the issue of differentiability of the pressure; proof that the spontaneous magnetization is always well defined; definition of first order phase transitions;
- 25/10/23: 1-dimensional Ising model; explicit computation of the pressure via the transfer matrix; comments on the paramagnetic behaviour of the 1-dimensional model for finite temperature; limit for 0 temperature; proof that the average magnetization concentrates on 0 under the Gibbs measure for $n \uparrow \infty$ and $h = 0$; introduction to infinite volume Gibbs states; definition of local function and of infinite volume state.
- 31/10/23: Definition of infinite volume Gibbs state; definition of translation invariant states; construction of two family of generators of local functions; introduction to correlation inequalities, GKS correlation inequalities with proof.
- 07/11/23: FKG correlation inequalities with proof and some consequences: Markov property type condition on boundary and monotonicity with respect to the volume.
- 08/11/23: Again on some consequences of correlation inequalities: extremal role played by the $\pm$ boundary conditions; proof of the existence of infinite volume Gibbs states with $\pm$ boundary conditions; introduction to phase diagram; probabilistic definition of first order phase transitions; statement of the main theorem on phase diagram; proof of some criteria for (non)-uniqueness of Gibbs states.
- 14/11/23: Definition of the critical inverse temperature; proof of the equivalence between analytic and probabilistic definition of phase transition; spontaneous symmetry breaking at low temperature; low temperature geometric representation; proof of $\beta_c(2) < \infty$ for the Ising model via the Peierls argument;
- 15/11/23: Extension to large dimensions to prove that $\beta_c(d) < \infty$ for any $d \geqslant 2$; uniqueness at high temperature, namely $\beta_c(d) > 0$ for any $d \geqslant 2$, via the high temperature representation; proof of $\beta_c(1) = \infty$;
- 21/11/23: Uniqueness in the regime of non-vanishing magnetic field; general comments on the complex analysis setting; Lee-Yang theorem (without proof); Lee-Yang circle theorem with proof using Asano contraction.